The Center
for
Photogrammetric Training

5 VERTICAL PHOTOGRAPHY

Photographic Scale Calculations Over Flat Terrain

Given: distance on the photograph and coordinates of corresponding points on the ground

$ab := 3.0833\,in$ $\qquad ab_m := ab \cdot 25.4\frac{mm}{in}$ $\qquad ab_m = 0.078\,m$

$X_A := 3910451.51\,m$ $\qquad Y_A := 244219.02\,m$

$X_B := 3910138.12\,m$ $\qquad Y_B := 243867.07\,m$

$$AB := \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \qquad AB = 471.256\,m$$

The denominator of the scale is (s):

$$s := \frac{AB}{ab} \qquad s = 6017$$

The scale of the photograph is 1:6017

Computing Flying Height on a Vertical Photograph

The basic equation for finding the flying height on a vertical photograph where the elevations of the points are not equal is

$$AB^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2$$

where AB is the ground distance between two points and X_i, Y_i are the ground coordinates of the two points at the end of the line. Recall that the ground coordinates can be determined from the photo measurements using the relationships:

$$X = \frac{H - h}{f}x \qquad Y = \frac{H - h}{f}y$$

where H is the flying height above the datum, h is the elevation of the ground point, f is the focal length of the camera, and x, y are the photographic coordinates of point i.

Substitute the values for X and Y into the distance equation yields

$$AB^2 = \left(\frac{H - h_B}{f} x_b - \frac{H - h_A}{f} x_a \right)^2 + \left(\frac{H - h_B}{f} y_b - \frac{H - h_A}{f} y_a \right)^2$$

Rearranging,

$$\begin{aligned} AB^2 &= \frac{1}{f^2}\left(Hx_b - h_B x_b - Hx_a + h_A x_a\right)^2 + \frac{1}{f^2}\left(Hy_b - h_B y_b - Hy_a + h_A y_a\right)^2 \\ &= \frac{1}{f^2}\left[\left(x_b - x_a\right)H + \left(h_A x_a - h_B x_b\right)\right]^2 + \frac{1}{f^2}\left[\left(y_b - y_a\right)H + \left(h_A y_a - h_B y_b\right)\right]^2 \end{aligned}$$

Let:

$$m = \left(x_b - x_a\right) \qquad p = \left(y_b - y_a\right)$$
$$n = \left(h_A x_a - h_B x_b\right) \qquad q = \left(h_A y_a - h_B y_b\right)$$

Then the ground distance can be represented by

$$\begin{aligned} AB^2 &= \frac{1}{f^2}\left(mH + n\right)^2 + \frac{1}{f^2}\left(pH + q\right)^2 \\ &= \frac{1}{f^2}\left[\left(m^2 + p^2\right)H^2 + \left(2mn + 2pg\right)H + \left(n^2 + q^2\right)\right] \end{aligned}$$

Moving the distance squared (AB^2) to the right hand side gives us the quadratic form of the equation. Let

$$a = \frac{\left(m^2 + p^2\right)}{f^2} = \frac{\left(x_b - x_a\right)^2 + \left(y_b - y_a\right)^2}{f^2}$$

$$b = \frac{2mn + 2pq}{f^2} = \frac{2\left(x_b - x_a\right)\left(h_A x_a - h_B x_b\right) + 2\left(y_b - y_a\right)\left(h_A y_a - h_B y_b\right)}{f^2}$$

$$c = \frac{n^2 + q^2}{f^2} - AB^2 = \frac{\left(h_A x_a - h_B x_b\right)^2 + \left(h_A y_a - h_B y_b\right)^2}{f^2} - AB^2$$

The solution is found by solving the quadratic equation in the form of

$$H = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Calculation of flying height over variable terrain
Direct Method

Given quantities: $f := 152.99$

$x_1 := 69$ $\quad y_1 := -174$ $\quad x_a := \frac{x_1}{60}\cdot 25.4$ $\quad y_a := \frac{y_1}{60}\cdot 25.4$

$x_2 := -28$ $\quad y_2 := 19$ $\quad x_b := \frac{x_2}{60}\cdot 25.4$ $\quad y_b := \frac{y_2}{60}\cdot 25.4$

$X_A := 3910451.5$ $\quad Y_A := 244219.0$ $\quad h_A := 287.8$

$X_B := 3909949.0$ $\quad Y_B := 243987.1$ $\quad h_B := 298.6$

Compute the ground distance

$$AB := \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \qquad AB = 553.4$$

Solve for the quadratic form

$$AB^2 = \left[\left(\frac{H - h_B}{f}\right)\cdot x_b - \left(\frac{H - h_A}{f}\right)\cdot x_a\right]^2 + \left[\left(\frac{H - h_B}{f}\right)\cdot y_b - \left(\frac{H - h_A}{f}\right)\cdot y_a\right]^2 \text{ solve}, H \rightarrow \begin{pmatrix} -636.5935358764271706 \\ 1215.263029639554608 \end{pmatrix}$$

Flying Height is 1215 m

Calculation of flying height over variable terrain
Iterative Method

Given quantities: $f := 152.997$

$x_1 := 69$ $\quad y_1 := -174$ $\quad x_a := \frac{x_1}{60} \cdot 25.4$ $\quad y_a := \frac{y_1}{60} \cdot 25.4$

$x_2 := -28$ $\quad y_2 := 19$ $\quad x_b := \frac{x_2}{60} \cdot 25.4$ $\quad y_b := \frac{y_2}{60} \cdot 25.4$

$X_A := 3910451.51$ $\quad Y_A := 244219.02$ $\quad Z_A := 287.86$

$X_B := 3909949.05$ $\quad Y_B := 243987.10$ $\quad Z_B := 298.64$

Compute photo distance and ground distance

$$ab := \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \qquad ab = 91.442$$

$$AB := \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \qquad AB = 553.401$$

$$h_{Avg} := \frac{Z_A + Z_B}{2} \qquad h_{Avg} = 293.25$$

Compute initial estimate of the flying height

$$H_1 := \frac{AB}{ab} \cdot f + h_{Avg} \qquad H_1 = 1219.2$$

Check the initial estimate of H:

$$X_A := \frac{H_1 - Z_A}{f} \cdot x_a \qquad Y_A := \frac{H_1 - Z_A}{f} \cdot y_a$$

$$X_B := \frac{H_1 - Z_B}{f} \cdot x_b \qquad Y_B := \frac{H_1 - Z_B}{f} \cdot y_b$$

$$AB_1 := \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \qquad AB_1 = 555.741$$

$$Diff := AB - AB_1 \qquad Diff = -2.34$$

Adjust the flying height

$$H_2 := \frac{AB}{AB_1} \cdot (H_1 - h_{Avg}) + h_{Avg} \qquad H_1 := H_2 \qquad H_1 = 1215.3$$

Check the current estimate of H:

$$X_A := \frac{H_1 - Z_A}{f} \cdot x_a \qquad Y_A := \frac{H_1 - Z_A}{f} \cdot y_a$$

$$X_B := \frac{H_1 - Z_B}{f} \cdot x_b \qquad Y_B := \frac{H_1 - Z_B}{f} \cdot y_b$$

$$AB_1 := \sqrt{\left(X_B - X_A\right)^2 + \left(Y_B - Y_A\right)^2} \qquad AB_1 = 553.411$$

$$Diff := AB - AB_1 \qquad Diff = -0$$